Axioms occupy a central position of the battlefield in many of the debates in the philosophy of mathematics. Mathematical knowledge is often separated into two tiers. Mathematical sentences such as '$5 + 7 = 12$' or the Pythagorean theorem are accepted as knowledge because they can be derived from axioms of arithmetic and geometry respectively. The axioms of arithmetic and geometry are valid because they are truths of set theory if interpreted as applying to the objects of arithmetic and geometry as encoded in sets. Thus, mathematical knowledge is often said be reduced to set theory (and pure logic). At the foundation of this explanation are the axioms of set theory. Axioms determine which structures are called 'sets' and thus, ultimately, which mathematical propositions are justified as knowledge.

Arguments for or against axioms can be divided into two categories. Intrinsic arguments appeal to base intuitions about our notion of a class or set. For example the Axiom of Extensionality, one of the least controversial of the standard axioms used by mathematicians, states that two sets are equal if and only if they contain the same members. Contrast extensional entities with intensional ones: *e.g.*, properties. We wouldn't say that the property $\phi_1(x) \stackrel{\text{def}}{=}$ 'x is the eldest son of the 41^{st} President of the United States' is equal to $\phi_2(x) \stackrel{\text{def}}{=}$ 'x is the President of the United States' even though both are true of the same individual. However we would say that the set $\{x|\phi_1(x)\}$ is equal to the set $\{y|\phi_2(y)\}$ because they both contain the same singleton element. An intrinsic argument for the Axiom of Extensionality follows from these intuitions: our notion of a 'set' is extensional not intensional, hence, the axiom is valid.

Extrinsic arguments appeal to the results of using the axioms. For example, an extrinsic argument for Extensionality is that intensional sets can be encoded as extensional ones but not vice versa. Extensional sets are more primitive than their intensional cousins and should be preferred, the argument goes (Maddy 1997, p. 40). In practice, intrinsic arguments alone cannot give us all of the axioms needed to found modern mathematics (Martin 1998, p. 229). In particular, certain axioms in ZFC[1] such as Infinity (there exists an infinite set), Empty Set (there exists a set with no elements) and others are directly motivated by a desire to found certain results in mathematics or avoid contradictions and not on any intuitions or claims of self-evidence. As intrinsic arguments alone leave us unable to account for the validity of all of the axioms required, the challenge for the philosopher of mathematics is to determine which extrinsic arguments constitute 'good grounds' for justifying axioms (Maddy 1998, p. 165).

In her most recent book, Penelope Maddy argues for a naturalised solution to this problem. Questions about the validity of axioms should be decided on mathematical, not philosophical grounds. On Maddy's account, attempts to justify our mathematical knowledge in any external arena separate or superior to mathematics (*e.g.* scientific, philosophical, etc.) are doomed from the start. By accepting a naturalist position, all of these questions are in principle already answered: an extrinsic justification of an axiom of set theory is valid if the mathematics that it generates is in line with the goals of the mathematician. The role of the philosopher of mathematics

[1] The Zermelo-Fraenkel axioms of set theory together with the Axiom of Choice. These axioms are generally accepted as standard.

is not to debate these extra-mathematical questions such as existence, truth and justification from a perspective external to mathematics. Rather, she is to identify the goals of the working mathematician and determine if mathematical methodology is the best way to reach those goals (Maddy 1998, p. 169). A critique of Maddy's naturalism is that it fails to give a normative account of mathematical knowledge. A naturalised philosophy of mathematics that retains stronger normative qualities is given by Quine. Maddy rejects Quine's arguments for placing mathematics in a continuous line with philosophy and the rest of science. A key part of her attack is the observation that Quine's naturalism doesn't provide any justification for the truths of pure mathematics. A retreat into a third, realist position (originally formulated by Maddy) may be the best way to address all of these concerns simultaneously.

Maddy's Mathematical Naturalism

They key concept behind Maddy's naturalism is that mathematics 'should be understood and evaluated on its own terms' (Maddy 1997, p. 184). In particular, Maddy argues that mathematics is separate from science and need not respond to pressures from scientific practice. This is a radical departure from Quine's naturalism (discussed below) which assigns to mathematics, science and philosophy the same status in our overall web of belief. In particular, Quine argues that mathematics is indispensable to science and that we should be committed to the existence of mathematical objects. Maddy rejects three key parts of Quine's account regarding mathematics. First, she questions Quine's criteria for ontological commitment by showing that scientists do not follow the patterns described by Quine. For example, though atom theory was an important part of scientists' best understanding of chemistry after 1860, it was not widely accepted by the scientific community until an experiment directly confirmed the existence of atoms in 1908. If Quine were right Maddy claims, scientists would have believed in the existence of the atom as soon as it became a crucial part of their best theory and would not have suspended belief until the results of the experiment (Maddy 1997, pp. 135-142). Second, she argues that mathematics is not a part of the philosophico-scientific continuum that Quine posits. Scientists do not treat themselves as the arbiters of mathematical existence; mathematics is used in science when it is convenient, and often it is understood to be an idealisation. It would be silly to believe in completed infinities simply because physicists use the infinite as a helpful idealisation in explaining *e.g.*, water waves (Maddy 1997, p. 144). Thirdly, Maddy rejects Quine's coupling of mathematics with science because it fails to give a satisfactory account of the branches of mathematics without application in scientific practice. In particular, Quine argues that we should accept new Axioms such as Constructibility (described in detail below) because it provides answers to independent questions in set theory and thus provides a simpler, more economical theory of sets (Quine famously prefers simple and austere theories). However consensus in the mathematical community is largely against extending ZFC with the axiom. Therefore, Quine's naturalism is unreliable on questions of abstract set theory according to Maddy (Maddy 1997, p. 106).

Maddy argues that mathematics is separate from science and thus, when we say that questions concerning mathematics should be answered naturalistically, we mean on mathematical, not scientific grounds. Maddy applies her naturalised methodology to an important question in abstract set theory: whether or not the Axiom of Constructibility should be adopted as an extension of ZFC. In the iterative conception of sets (Boolos 1998, p. 19), each stage $\alpha + 1$ in an ascending hierarchy contains all of the sets that can be formed from combinations of sets in α. At the base are the singleton sets formed from individuals. The next stage contains the sets formed from all of the sets present at the base level and individuals. This universe of sets[2] is referred to in the literature as V and is just one model of the sets generated from the standard axioms, ZFC. A second way of conceiving sets is the constructible model L. Here, rather than combining all of the sets from stage α in forming the sets at stage $\alpha + 1$, $L_{\alpha+1}$ contains only the sets that can be formed by a first order proposition with quantifiers ranging over the sets in L_α. The Axiom of Constructibility asserts that $V = L$ (Maddy 1990, p. 65). It is easy to see that $V = L$ escapes any intrinsic justification. If we are to have good grounds for adding it to our standard axioms, it will have to be justified extrinsically.

Maddy takes consistency as an implicit goal in set theory: if adding an axiom allowed one to introduce contradictions that could not be deduced from a more conservative theory alone, we would reject the axiom. But here, consistency is no guide. Gödel showed that if ZFC is consistent, then ZFC + $V = L$ is consistent. Further, $V = L$ is useful because it settles independent questions in set theory, including

[2] I use the phrase 'universe of sets' as a way to refer to the collection of structures that can be generated using the standard axioms. It is not meant to imply a specific ontological position.

Cantor's Continuum Hypothesis. However, despite these virtues (which led Quine to argue for the axiom), most set theorists have rejected Constructibility because it limits the universe of sets to definable entities. The consensus is that a stage theory that forms sets combinationally will be richer than one where they are limited to well-defined propositional formulas (Maddy 1997, p. 84). If we accepted Constructibility, we would restrict the universe of sets.

Maddy extrapolates from such examples two maxims, Maximise and Unify, to serve as guides for extrinsic justifications of axiom candidates. Given that the primary goal of set theory is to provide a foundation for the rest of mathematics, Maddy argues that these maxims guide our choices about axioms to ones that further that goal. Maximise tells us, 'set theory should not impose any limitations of its own: the set theoretic arena in which mathematics is to be modelled should be as generous as possible' (Maddy 1997, p. 210). Unify keeps set theory as primitive as possible, thus allowing it to serve as a single foundation for mathematics (Maddy 1997, p. 209). Consistency is taken to be an obvious, implicit goal of set theory. According to Maddy, the practice of set theory is constrained by these maxims; they determine good grounds for accepting or rejecting axiom candidates. In the case of Constructibility, Maximise instructs us to reject the axiom because it limits the universe of sets.

Maddy's refusal to fit mathematics into the scientific justificatory framework has two important consequences. First, it frees her from defending any form of realism. When Quine places mathematics at the centre of our best theory for forming reliable beliefs (science), he is impelled to believe in the existence of mathematical entities.

For Maddy, mathematics is helpful in science only as a useful fiction. It is not an inseparable part of science and thus is not subject to the same ontological commitment as other entities implied by the rest of our scientific theory. Second, it relieves mathematics from responding to pressure from science and instead installs it as master of its own domain. Specifically, only mathematical arguments can be used to support or critique a particular choice in mathematical methodology. Arguments which appeal to extra-mathematical factors including scientific and philosophical concerns are irrelevant.

Quine's Naturalism

Quine's naturalism is rooted in the notion that philosophy is continuous with science (Quine 1951, pp. 42-43). Philosophical questions are not prior or superior to scientific ones, but must be posed and evaluated within the scientific framework. Quine's naturalism was motivated in response to Carnap's positivism. Carnap noticed that many questions about science and mathematics are external to the working languages of those disciplines. On his view, sentences such as 'the number 5 exists' are meaningful only if they are taken inside the linguistic framework of mathematics (Carnap 1983, pp. 242-243). In that framework, the truth of sentences such as the above are easily answered (in this case, affirmatively). If we ask the same question outside of the linguistic framework of mathematics, we are asking a pseudo-question because we are operating outside of a linguistic and evidentiary framework in which to evaluate it.

Quine moderates Carnap's strict separation of internal and external questions that reduces many philosophical enquiries to pseudo-questions. Quine argues that many philosophical questions can be evaluated within the linguistic framework of science. For example, on ontological questions, Quine instructs us to be committed to entities that are an indispensable part of our best overall scientific theory. Thus on Quine's view, I should believe in the existence of physical objects such as the cup in front of me because believing in physical objects has proved useful to my best theory for understanding the world. I should also believe in things that are more difficult to observe. Forces, atoms, quarks and sets are all important to our best scientific theory and thus have the same ontological status as medium-sized physical objects (Quine 1951, p. 42). An important part of this account is the notion of *holism*: a scientific theory is tested as a single piece. If the theory is confirmed, then all of its constituent parts are given confirmation; if it fails then any part of the theory (including mathematics) can be modified to account for the discrepancy between theory and observation. He sees mathematics as indispensable to our best theory about the world and hence gives the entities used in applied mathematics the same status as the entities of physics, chemistry and the rest of science.

Normativity and Naturalism

A weakness of Maddy's naturalism is that it reduces philosophy of mathematics to mere description of the practices of mathematicians. She replaces the traditional work of philosophers with two tasks. In order to argue for a particular mathematical method (axiom), the naturalist identifies the goals of mathematics, and shows that the method is the most effective means for achieving the goals. In the case of set theory, this amounts to extrapolating maxims such as Maximise and Unify, and arguing that these are the best ways to further the goals of set theory. Maddy admits that the first part of this naturalised inquiry, identifying goals, is largely descriptive. However the second task, arguing that the method in question is optimal, goes beyond pure description, she says. Whether a method is optimal is 'an objective matter, about which individual practitioners, and even the entire community, could be mistaken' (Maddy 1997, p. 170). Thus, Maddy claims, because the naturalist entertains the possibility that mathematicians are mistaken, she must depart from sociology and description. However, it is not clear that this is the case. The only access that the philosopher has to the goals of mathematicians is through an examination of their methods. She has no direct access to the goals and no external perspective from which to consider the objective question. Hence, the goals she identifies are entangled with the methods she observes. In practice, the Maddian naturalistic philosopher is unable to unravel goals from methods and is left in the position of offering 'rubber stamp' approval to the methods of working mathematicians (Colyvan 2001, p. 97).

Maddy's account fails to give an explanation of mathematical knowledge with strong normativity. One might expect the concept of mathematical truth to provide normative guidance. In epistemology, the conditions for having knowledge are generally taken as being in possession of justified true belief (what counts as a justification or truth is debatable). While beliefs are essentially internal and potentially dependent on purely mental factors, our idea about what counts as knowledge is guided by an external condition: that the belief be true. Truth provides a measure of normativity in our concept of knowledge. However in Maddy's naturalistic account truth is no guide. For her, a high-level mathematical belief is true if it is derivable from the standard axioms. Saying that an axiom is 'true' is just a shorter way of saying that the axiom 'serves the ends of set theory better than set theory without it' (Maddy 1997, p. 168). As argued above, the philosopher of mathematics has no external perspective from which to evaluate how well the methods of mathematicians serve the goals. Hence, the factors that ultimately shape the universe of sets and the collection of true mathematical formulas are the goals of set theory as extrapolated from methodology. Truth offers no further degree of normativity for the Maddian naturalist.

Stretched to a slightly more radical conclusion, Maddy's naturalism could be used to justify the claim that almost any mathematical proposition is true. Timothy Gowers, a self-described naturalist mathematician, sees no problem with using certain axioms when they are convenient and discarding the same axioms when they give 'bizarre' results (Gowers 2002). He is happy to note that the Axiom of Choice is assumed to be true or false in a proof without further worry about the implicit contradiction if mathematics uses both Choice and not Choice in the discipline as a whole. Accepting Choice and not Choice in valid mathematical proofs goes against Unify (we now have two disjoint theories of sets) and consistency, which Maddy thinks is always an overarching goal of mathematics. More importantly, though the mathematician thinks they know when it is right to use Choice, and when it may be assumed false, we aren't given any description of how these decisions are made in any principled manner. For a proper normative account, we need some criteria for evaluating when a result is 'bizarre' or when it fits with 'mainstream mathematics'. Is the Maddian naturalist willing to say that the goals of mathematics are not consistency, Maximise and Unify but rather, as Gowers's position suggests, to generate any interesting proof? And if so, who decides what is interesting versus uninteresting, or when to accept or reject an axiom such as Choice in a proof?

In contrast to Maddy's largely descriptive characterisation of a naturalistic philosophy of mathematics, Quine's naturalism does provide a measure of normativity to our understanding of mathematical knowledge. Quine's position is that mathematics is an indispensable part of our best scientific theory, a strand in our web of beliefs about how the world works. By treating mathematics as continuous with science, it both exerts and responds to pressures as the entire theory is repeatedly tested by experiences. When our hypotheses about the world fail to square with experience, we adjust the hypotheses in order to make them better fit with what we have observed. Using this argument, Quine suggests, 'there is no clear boundary between theoretical physics and mathematics' (Quine and Ullian 1978, p. 81). In as much as both do a good job of explaining our experiences in the world, we accept them. When they go wrong, we adjust our hypotheses to make them fit our observations. While certain hypotheses, such as core mathematical truths, may be so foundational and general that we would be more willing to make drastic revisions to physics than to mathematics, in principle at least, mathematics is open to revision within our web of belief on Quine's account.

Quine's story about mathematical objects tells us that we should be committed to their existence because they are indispensable to our best scientific theory. Further, his account provides some normativity because mathematical hypotheses, like all other scientific theories, are refutable and revisable. However, in practice, mathematical hypotheses are rarely adjusted on Quine's approach. The connections between theories in Quine's web of belief are purely logical and the only requirement is that the overall theory be coherent. If we discover a discrepancy between our theory and observations, we are much more likely to modify a scientific hypothesis at the edge of the web than a mathematical one in the centre. Changing a belief at the centre would imply changes to many theories in the web. Thus, the easiest way to recover a coherent theory is usually to adjust a hypothesis that lies closer to our observations: *i.e.*, a specific scientific theory. The stronger normativity offered to mathematical knowledge on Quine's approach rarely comes into play.

Colyvan's Defence of Indispensability

We've seen that Maddy's naturalism differs from Quine's in that it separates mathematics from science and thereby reduces the philosophy of mathematics to the description of mathematical methodology. Quine's account gives us a framework in which mathematical knowledge can be justified on normative criteria as it responds to logical pressures from other parts of our overall theory. However, we still have Maddy's arguments rejecting Quine's indispensability argument as applied to mathematics. Mark Colyvan claims that Maddy's objections to Quine's account – that it fails to explain the beliefs of working scientists (*e.g.*, regarding the status of the atom), that mathematics is used indiscriminately as a helpful idealisation in science without regard to the ontological commitment required by Quine, and that it fails to provide any explanation for pure mathematics – can be countered with a few small adjustments to our understanding of naturalism.

Colyvan's answer to the first objection is to insist that the naturalist's position regarding ontological commitment is correct; scientists who refuse to believe in the existence of entities implied by their best theory are mistaken. In the case of atoms, scientists who questioned whether or not atoms existed after they became indispensable to chemistry but before they were directly confirmed by experiment were being intellectually dishonest. Rather then finding fault with naturalism, we can argue that scientists were mistaken. Further, this kind of critique can be framed within the bounds of science; we need not place the philosopher in a privileged position. Maddy seems to argue that naturalism impels the philosopher to give on any issue where practice and philosophy clash. Colyvan suggests that this is not the case: 'there is much ground between a first philosophy, which Quine rejects, and the rubber-stamp role, which Maddy seems to advocate' (Colyvan 2001, p. 97). Maddy's objection that Quinean naturalism fails to accurately describe the ontological commitment of scientists does not go through if we understand philosophy as being truly continuous with science and not completely subservient to it.

The second objection, that scientists use mathematics as a convenient idealisation without regard to the ontological status of mathematical objects implied by their inclusion in a theory is answered by Colyvan on similar grounds. In cases where the idealisation is blatantly false (an infinitely deep ocean in the study of waves) he notes that we can disregard the ontological commitment implied by these assumptions because the theory is not understood as literally true (Colyvan 2001, pp. 101-102). In cases where no blatantly false assumption is used, such as the use of real numbers in modelling space-time, we can regard the question as open whether space-time is actually continuous. If we discover that it is not, but that reals are still useful as an idealisation in performing calculations, then we understand the continuum as just another useful idealisation but not literally true. He goes on to argue that scientists are in fact discriminating when it comes to introducing potentially problematic mathematics into their theories. He cites the intense debates over entities such as complex numbers as evidence that scientists do care about the ontological status of controversial mathematical objects when forming their best theories.

The most problematic objection that Colyvan confronts is the claim that Quine's argument fails when applied to pure mathematics. Regions of mathematics, such as abstract set theory, that have no application to our best theory for understanding the world are offered no support by Quine's naturalism. Colyvan gives three responses to this objection. First, history is full of cases where supposedly pure mathematical theories found surprising applications in a scientific theory. Thus, the pure mathematician is in the same position as the theoretical physicist who posits theories without any clear application in sight. Second, even if certain areas of set theory that currently have utility to science were shown to be idealisations, the abstract set theorist could be said to be doing mathematical recreation. The abstract set theorist would suspend their ontological commitment to the entities generated by their practice while still continuing to investigate the landscape of set theory recreationally. Finally, the reason that set theorists go about their practice without regard to applications in the sciences is not because it is fundamentally different from science, but rather because of a division of labour. Colyvan characterises set theory as the 'most global area of the most global theory'. Thus, practitioners are too far removed from the consequences of their work to consider the ramifications of *e.g.*, Constructibility to physics and other scientific fields. These three similar responses amount to a trickle-down policy regarding pure mathematics: applied mathematics is indispensable to science so pure mathematicians are given freedom to recreate, looking towards future applications in science.

Colyvan rescues Quine's application of the indispensability argument regarding mathematics from Maddy's critique. However, in the end his position is essentially that of a Gödelian Platonist. In particular, he rejects the notion that mathematical objects are causally active though he admits that mathematical knowledge on the Quinean approach is empirical and thus *a posteriori* and perhaps contingent (Colyvan 2001, p. 143). Quine and Colyvan both argue that mathematical objects are abstract entities (Quine 1960, pp. 266-270). As we shall see in the next section, the claim that mathematical objects are concrete, causally active entities gives us the strongest degree of normativity in a naturalistic philosophy of mathematics. Colyvan's rejection of a physicalistic explication of mathematical entities leaves his philosophy of mathematics open to the same critiques of unreliability that Quine's suffers regarding pure mathematics.

The Third Way

The two pictures of naturalism presented by Quine/Colyvan and Maddy portray the nature of mathematical knowledge very differently. For Quine, mathematics is just another part of our holistic theory of the world, responsive to pressures within that system. Maddy separates mathematics from science and roots justification of our mathematical knowledge in the goals of mathematicians. I've argued that Maddy's view prohibits all philosophical and scientific considerations from influencing questions about mathematical knowledge and leaves us, ultimately, with a largely descriptive account of mathematics. However, while Colyvan offers reasonable solutions to the problems with Quinean naturalism noted by Maddy, the rescued Quinean account still fails to explain what guides pure mathematics. The indirect pressures exerted by applied mathematics on abstract parts of the discipline are weak at best. While Quine's account provides a normative conception of mathematical knowledge for the applied parts of mathematics, it leaves us in the same boat as Maddy with regard to abstract set theory. Quine's unreliability on questions such as whether we should accept Constructibility as an axiom is damning evidence that the increased normativity offered by his view is of no practical benefit when we wander too far from applied mathematical investigations.

A third flavour of naturalism which doesn't suffer from either of the above critiques can be obtained by placing mathematical objects, as Maddy did in her earlier position as a realist, in the physical universe. The realist Maddy argued that we use our ordinary physical senses to perceive sets: when I look at a cup resting on a table I actually perceive the individual cup, the set containing the cup, the set containing the cup and the table and so on (Maddy 1990, pp 50-51.). On this account, our axioms of set theory are justified in so far as they conform to the real world of sets. The challenge to the set theorist is to find axioms that accurately describe the actual world of sets. Here, set theory is just a special branch of science that investigates another area of the physical, causal universe. Questions such as whether the Axiom of Constructibility is true can be answered by appealing to our current best hypotheses about sets. If we later discover by some empirical means that sets are in fact constructible, then we revise our theory, in this case the standard axioms, to accommodate this change. The distinction between pure and applied questions in set theory disappears on this realist account. Set theoretic realism gives the desired normativity to all fields of mathematics, applied and unapplied, and firmly binds mathematics to science. Mathematical investigation is moved from the centre of our web of belief to the exterior. It is not merely responsive to logical pressures as a part of a coherent theory. Instead, direct causal observations can be used to confirm or refute our best mathematical hypotheses (axioms).

What we want from an epistemological account of mathematics is an explanation of why mathematical beliefs that count as knowledge are reliable. Platonism fails because it offers no explanation of how we could have any knowledge about causally inert abstract entities. Maddy's naturalism fails because it does not offer an account of what actually guides mathematical practice beyond the whims of the mathematician and reduces too easily to mere description. Quine's naturalism provides a notion of normativity for applied mathematics, but falters when stretched to the far reaches of abstract set theory. Only the set theoretic realist account succeeds where the others fail. By placing mathematical entities in the physical universe, it allows mathematics to be conceived of as part of our overall best scientific theory. Thus, the axioms of set theory are just scientific hypotheses, subject to confirmation and refutation by causal means along with the rest of our web of beliefs. This style of naturalised realism does the best job explaining both how we have mathematical knowledge and why it is reliable (see Benacerraf (1983) for a discussion of why achieving both of these simultaneously has been difficult for most philosophies of mathematics). In the end, mathematics is best understood as just another plank in Neurath's boat. As Quine was fond of quoting, we must 'rebuild [our] ship on the open sea, without ever being able to dismantle it in dry-dock and reconstruct it from the best components' (Neurath 1932). We cannot rip out the centreboard (as Maddy would have us do) and expect to steer a straight path.

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